Analyzing Signals

Fourier transform
- frequency content
- linear combination of $\sin(\omega t)$ and $\cos(\omega t)$

Spectrum

spatial domain  frequency domain
Spectrum

spatial domain  frequency domain

Spectrum

spatial domain  frequency domain
Localized Analysis

Gabor (1940)
- time frequency analysis
- windowed Fourier transform
Gabor Transform

Find
- frequency $\omega$ in the vicinity of $b$

$$F(b, \omega) = \int f(x) g(x - b) \sin(\omega x) \, dx$$

window function at $b$

at frequency $\omega$

Gabor Transform

Spatial domain

Gabor domain

b
Gabor Transform

Problems
- discrete version very difficult to find
- no fast transform
- fixed window size!

Solution
- large windows for low frequencies
- small windows for high frequencies

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Gabor Transform

Gabor bases  Wavelet bases
Wavelets

Translates and dilates of one function

\( \psi \left( \frac{x - b}{a} \right) \)

Mother wavelet
- local in space
- local in frequency
  - smooth: no high frequencies
  - integral zero: no low frequencies

Wavelets

Wavelet bases Spectra
Wavelet Transform

Find

- scale a at location b

\[ F(a, b) = \int f(x) \psi \left( \frac{x - b}{a} \right) dx \]

Wavelet Transform

Spatial domain Wavelet domain

\[ 1/a \]
Summary

Fourier analysis
- global frequency properties

Picking out local phenomena
- windowed Fourier transform: Gabor

Wavelets
- window varies with frequency

Making it Practical

A simple example
- Haar transform

Building more powerful transforms
- Lifting scheme

Generalizations
- making it work on general domains